SYNERGETIC APPROACH TO A HARMONIC DISTURBANCE OBSERVER SYNTHESIS FOR THE AMPHIBIAN MOTION CONTROL SYSTEMS

Nguyen Phuong,
University of Technical Education Hochiminh City

ABSTRACT

The aircraft amphibian, as a control object (CO), has an extremely complex structure consisting of a set of the subsystems including the exchange processes of force, energy, matter and information. This control object operates in the complex environments as an atmosphere as well as an adjoining surface of water and air. The problem is to design a regulator that provides control capabilities of the flight modes with impact on the surrounding environment. The requirement to designed regulator is quick responsibility to adapt to an impact of chaotic disturbances of the environment. In this report we consider a synthesis method of nonlinear control system for the aircraft amphibian motion with the state observers of the harmonic disturbances based on the synergetic approach in modern control theory.

KEY WORDS: Synergetics, system synthesis, regulator design, chaotic disturbances, aircraft amphibian.

1. INTRODUCTION

The solution of the various control tasks based on using a control object state vector. In real conditions of full state vector measurement for a reason is not feasible. For this purpose, the control system introduces a subsystem of state estimation - a state observer.

For linear systems, it is distinguished three kinds of observers: a full-order state observer (Kalman Observer), which has a dimension of a state vector like the control object; a reduced order observer (Luenberger Observer) and an increased order observer (adaptive observer) [1, 2]

Proposed in this article, the nonlinear observer could be referring to a reduced order observer. Even more challenging is a problem of estimating unmeasured external disturbances. A basic idea of the perturbation estimation is as follows: It is necessary to construct a model of external influences, which is in the form of a homogeneous differential equation system with known coefficients and unknown initial conditions. The model is combined with the perturbation model and with this received enhanced system observer is constructed. Obtained with it estimates include the estimates of object state variables, and evaluation of external influences.

The asymptotic observer design methods are applicable for a wide class of the nonlinear systems proposed in [3, 4, 5]. In this work, a new version of an amphibian control methods and problems, which are solved by the dynamic synergetic regulators to such observers, is described. These observers have carried out an unmeasured harmonic external disturbance evaluation effecting on the amphibian. The nonlinear external perturbation observers (NEPO) consist of a monitoring contour and a control circuit that operates in parallel.

2. THE PROBLEM STATEMENT

Suppose that a behavior of the control object and external disturbances effecting on it could be described by a differential equations system:

\[ \dot{x} = g(x, z, u); \]
\[ \dot{z} = h(x, z, u). \]
Where: vectors \( x, z \) are state vectors; \( u \) is a control vector; functions \( g(\cdot) \) and \( h(\cdot) \) are continuous nonlinear functions. Assuming that vector \( x \) is observable, but vector \( z \) is unobservable.

Then a observer synthesis problem can be formulated following. We need to synthesize NEPO in form of:

\[
\dot{w}(t) = R(x, w); \\
\dot{\hat{z}}(t) = K(x, w),
\]

where \( w \) is an observer state vector; \( \hat{z} \) is evaluation vector of unmeasured external disturbances.

Then the synthesized NEPO must provide:
- a closed system asymptotic stability;
- a stabilization of the pitch angle, altitude and flight speed;
- an assessment of unobserved external perturbations;
- a compensation of external disturbances.

The synthesis procedure of NEPO is divided into three stages:
- The first is a synthesis of control laws \( u_i \) that ensure a implementation of the required technological problems (assuming that all state variables of the control object are observable);
- The second is a synthesis of an observer for unobservable state variables and the unmeasured disturbances.
- The last is a replacement of unobservable variables in the synthesized controls by their evaluations.

3. **THE OBSERVER SYNERGETIC SYNTHESIS PROCEDURE**

This section describes the synergetic synthesis procedure of control laws for the amphibian longitudinal motion that is affected by disturbances in the form of harmonic waves.

3.1 **A synergetic synthesis procedure of control laws \( u_i \)**

A common model of a CO’s space movement is presented by the 12th order differential equation system through the Euler angles. A movement on water or a taking-off is rational to consider the longitudinal motion model:

\[
\begin{align*}
\dot{x}_1(t) &= b_1 x_1 x_2 - g \sin x_3 + a_1 (P_e - F_{aw} - F_{aw}) + M_1(t); \\
\dot{x}_2(t) &= b_2 x_2 x_3 - g \cos x_3 + a_2 (P_i + F_{aw} + F_{aw}) + M_2(t); \\
\dot{x}_3(t) &= a_3 (M_1^h + M_2^h) + M_3(t); \\
\dot{x}_4(t) &= x_3 \sin x_1 + x_2 \cos x_1; \\
\dot{x}_5(t) &= x_1; \\
\dot{x}_6 &= x_1 \cos x_1 - x_2 \sin x_1.
\end{align*}
\]

Where: \( x_1, x_2 \) are projections of the velocity vectors \( V_x, V_y \) on corresponded the intertwined coordinate system axes; \( x_3 \) is a longitudinal angular velocity; \( x_4, x_5 \) are coordinate projections of the s of CO’s gravity center \( x_c, y_c \) on corresponded axes \( Ox \) and \( Oy \); \( x_6 \) is pitching angle \( \theta \); \( m \) is CO’s weight; \( m_x = (1 + \lambda_1) m \), \( m_y = (1 + \lambda_2) m \) are the CO’s «attached» weights; \( F_{aw}, F_{aw} \) are projections of total vectors of aerodynamic forces on corresponded with the intertwined coordinate system axes \( Ox \) and \( Oy \); \( F_{aw}, F_{aw} \) are total vector projections of hydrodynamic and hydrostatic forces on corresponded with the intertwined coordinate system axes \( Ox \) and \( Oy \); \( M_1^h, M_2^h \) are longitudinal aerodynamic moment and longitudinal moment formed by hydrodynamic and hydrostatic forces; \( M_3(t) \) are disturbances; \( a_1 = m_x^{-1} ; a_2 = m_y^{-1} ; a_3 = I_z^{-1} ; \)

\[
b_1 \frac{m_y}{m_x}; b_2 = \frac{m_x}{m_y}.
\]

In control processes, a CO’s longitudinal motion elevator, flaps and engine thrust
control lever are the active control organs (fig. 1). A technical solution, that provides effectively basing and operating of the aircraft on the water surface, is to determine its shapes – a seaplane aerodynamic scheme. Consequently, controls in the model (2) will be an engine thrust, depending on the deviation of the engine thrust control lever; a total of aerodynamic forces and a total of longitudinal moment, depending on changes in the flaps and a elevator deflection.

For control of CO’s longitudinal motion there are some strategies: controlling individual channels or all channels simultaneously. Of course that a vector strategy requires a more complex algorithm structure of a regulator, but it allows a more flexible three-channel CO control.

The control problem of a longitudinal motion is finding a control vector $u_1 = P_x - F_{ax} - F_{cx}, \ u_2 = P_y + F_{ya} + F_{cy}, \ u_3 = M_{za} + M_{ze}$ – are the control acts.

For the model (3), a task goal is an implementation of the desired invariants (2), we formulate the first set of macro-variables $\psi_1, \psi_2, \psi_3$,

$$\psi_1 = x_1 - V_0,$$

$$\psi_2 = x_2 - \phi_1 (x_4, x_5, z_1, z_2, z_3);$$

$$\psi_3 = x_3 - \phi_2 (x_4, x_5, z_1, z_2, z_3),$$

which must satisfy a solution of the following functional equations:

$$T_i \psi_i (t) + \psi_i = 0, \ T_i > 0, \ i = 1...3; \ (5)$$

At an intersection of the invariant manifolds, $\psi_i = 0, i = 1...3$, there is a dynamic “phase space compression” phenomenon, and dynamics of a closed-loop system will be described by the decomposed model:

$$\dot{x}_4 (t) = V_0 \sin x_5 + \phi_1 \cos x_5;$$

$$\dot{x}_5 (t) = \phi_2;$$

$$\dot{x}_6 (t) = V_0 \cos x_5 - \phi_1 \sin x_5; \ (6)$$

Now we introduce a second set of macro variables:

$$\psi_4 = x_4 - H_0; \ \psi_5 = x_5 - \theta_0. \ (7)$$

A set of macro variables, that was introduced by (7), must satisfy a solution of the functional equation systems:

$$T_i \psi_i (t) + \psi_i = 0, \ T_i > 0, \ i = 4, 5; \ (8)$$

For determining “inner” controls $\phi_1, \phi_2$ in form of a function depending on state variables, we solve jointly the equations from (6) to (8), and receive:

$$\phi_1 = - \frac{T_4 V_0 \sin x_5 + x_4 - H_0}{T_4 \cos x_5}; \ \phi_2 = \frac{-x_4 + \theta_0}{T_5}. \ (9)$$
Further the external control vectors $u_i$ are found by solving simultaneously the functional equation systems (4) and the equation model (1):

$$u_i = \frac{1}{a_i} \left( g \sin x_5 + - \frac{x_1 + V_0}{T_1} - z_i \right);$$

$$u_2 = Ax_1 + Bx_2 + Cx_3 + Dx_4 - \frac{1}{a_2} z_2 + E;$$

$$u_3 = -\frac{1}{T_3 T_5 a_3} \left( (T_3 + T_5)x_3 + x_5 - \theta_0 \right) - \frac{z_3}{a_3}$$

(10)

Where we indicate: $A = -\frac{\sin x_5}{T_4 a_2 \cos x_5}$;

$$B = -\frac{T_2 + T_4}{a_2 T_2 T_4};$$

$$C = -\frac{x_5 \sin x_5}{a_2 T_4 \cos^2 x_5} + \frac{H_0 \sin x_5 - T_4 V_0}{a_2 T_4 \cos^2 x_5};$$

$$D = -\frac{1}{a_2 T_2 T_4 \cos x_5};$$

$$E = \frac{H_0 - T_4 V_0 \sin x_5}{a_2 T_2 T_4 \cos x_5} + \frac{g \cos x_5}{a_2}.$$

Whereas the synthesized control laws of object (1) \{ $u_1$, $u_2$, $u_3$ \} provide an implementation of the required technological problems, it is necessary to go to a description of the observer synthesis procedure.

3.2 The synthesis procedure of non-linear state observer

According to a method of Analytical Design of Aggregated Regulators (ADAR), created by an Russian Scientist – A.A. Kolesnikov, during a synergetic synthesis procedure of observers, it is necessary to use the following extended system model of the control object (11) [3, 4]:

$$\dot{x}_1(t) = -g \sin x_5 + a_1 u_1 + z_i;$$

$$\dot{x}_2(t) = -g \cos x_5 + a_2 u_2 + z_2;$$

$$\dot{x}_3(t) = a_3 u_3 + z_3;$$

$$\dot{x}_4(t) = x_1 \sin x_5 + x_2 \cos x_5;$$

$$\dot{x}_5(t) = x_3;$$

$$\dot{x}_i(t) = x_i \cos x_5 - x_i \sin x_5;$$

$$\dot{z}_1(t) = s_1; \dot{s}_1(t) = -\sigma_i^2 z_i;$$

$$\dot{z}_2(t) = s_2; \dot{s}_2(t) = -\sigma_i^2 z_2;$$

$$\dot{z}_3(t) = x_3; \dot{s}_3(t) = -\sigma_i^2 z_3;$$

Where $\sigma_i$ – harmonic disturbance angular frequencies, $z_1, z_2, z_3$ are projections respectively of an indignant linear, longitudinal and an angular accelerations.

The last six equations of the system (11) is a dynamic model of harmonic disturbances, and variables $z_1, s_1, i=1..3$ are state variables.

An observer design of state variables is based on principles of a synergetic approach in control theory, more exactly on the ADAR method, which is described in works [3, 4]. In particular case, when $\dim \psi(t) = 1$, an expression

$$\dot{\psi}(t) = L(y) \psi$$

(12)

could be presented in the following form:

$$\dot{\psi}_i(t) + L_i \psi_i = 0, \quad L_i > 0.$$

(13)

Now we conduct a synthesis of observers for the object (1). Let put $y = [x_i]_{i=1,...,5}$, $v = [z_j, s_j]_{j=1,2,3}$. We determine an assessment of state variables $z_i, s_i$, it is necessary to choose forms of $\psi_1, \psi_2$ like:

$$\psi_1 = \beta_{i1} (z_i - \hat{z}_i) + \beta_{i2} (s_i - \hat{s}_i);$$

$$\psi_2 = \beta_{21} (z_i - \hat{z}_i) + \beta_{22} (s_i - \hat{s}_i)$$

(14)

where $\beta_{ij} \neq 0$ are constants and $\beta_{11} \beta_{22} - \beta_{12} \beta_{21} \neq 0$. In this, valuations $\hat{z}_i, \hat{s}_i$ of state variables $z_i, s_i$ could be formed by

$$\hat{z}_i = f_1(x_i) + w_1;$$

$$\hat{s}_i = f_2(x_i) + w_2.$$

(15)
where \( f_1(x_i), f_2(x_i) \) are unknown functions. Then putting (14) into an equation in the form of (13):
\[
\dot{\psi}_i(t) + L_1 \psi_i = 0, \quad L_1 > 0;
\dot{\psi}_j(t) + L_2 \psi_j = 0, \quad L_2 > 0,
\]
That subjects to the equations (15), we receive:
\[
\begin{align*}
\beta_1 & \frac{dx_1}{dt} - \frac{\partial f_j(x_i)}{\partial x_i} - \frac{d \psi_i}{dt} + \\
\beta_2 & \frac{dx_2}{dt} - \frac{\partial f_j(x_i)}{\partial x_i} - \frac{d \psi_i}{dt} + \\
& + L_1 \left[ \beta_1 \left( z_i - f_j(x_i) - w_i \right) + \beta_2 \left( s_i - f_j(x_i) - w_j \right) \right] = 0,
\end{align*}
\]
(17)
With the equations (17) subject to the object equations (11), we receive:
\[
\begin{align*}
\beta_1 & \frac{dz_1}{dt} - \frac{\partial f_j(x_i)}{\partial x_i} \left( -g \sin x_i + a_{ij} + z_i \right) - \frac{dw_i}{dt} + \\
\beta_2 & \frac{dz_2}{dt} - \frac{\partial f_j(x_i)}{\partial x_i} \left( -g \sin x_i + a_{ij} + z_i \right) - \frac{dw_i}{dt} + \\
& + L_1 \left[ \beta_1 \left( z_i - f_j(x_i) - w_i \right) + \beta_2 \left( s_i - f_j(x_i) - w_j \right) \right] = 0;
\end{align*}
\]
(18)
In the equations of the observer (18) it should not be presented unobserved coordinators \( z_i, s_i \). In order to exclude them out of the system, it’s necessary to choose:
\[
\begin{align*}
f_1(x_i) &= \frac{\beta_{12}}{\beta_1} \beta_{21} \beta_{22} \left( \beta_{12} \beta_{21} - \beta_{22} \beta_{21} \right) x_i, \\
f_2(x_i) &= \left( \beta_{21} \beta_{22} \beta_{21} \beta_{22} \beta_{21} \beta_{22} \beta_{21} \beta_{22} - \sigma_i \right) x_i,
\end{align*}
\]
(19)
\[
L_i = -\frac{\beta_1}{\beta_12} > 0, \quad L_2 = -\frac{\beta_{21}}{\beta_{22}} > 0
\]
with that to solve the equation system (18), we found:
\[
\dot{\hat{w}}_1 = \left[ \frac{\beta_{11}}{\beta_{12}} \right] + \sigma_i \left[ \beta_{21} \beta_{22} \beta_{21} \beta_{22} \beta_{21} \beta_{22} \beta_{21} \beta_{22} \right] x_i + \\
+ \left[ \beta_{21} \beta_{22} \beta_{21} \beta_{22} \beta_{21} \beta_{22} \beta_{21} \beta_{22} \right] \left[ \beta_{11} + a_{ij} - g \sin x_i \right] + w_i
\]
(20)
and valuations \( \hat{z}_1, \hat{s}_1 \) of state variables \( z_1, s_1 \) will be:
\[
\hat{z}_1 = \left[ \beta_{11} \beta_{22} \beta_{21} \beta_{22} \beta_{21} \beta_{22} \beta_{21} \beta_{22} \right] x_i + w_i,
\]
\[
\hat{s}_1 = \left[ \beta_{11} \beta_{22} \beta_{21} \beta_{22} \beta_{21} \beta_{22} \beta_{21} \beta_{22} \right] - \sigma_i x_i + w_i
\]
(21)
Similarly, to define estimations \( \hat{z}_2, \hat{s}_2 \), of state variables \( z_2, s_2 \), we choose the following macro variables
\[
\psi_3 = \beta_{33} (z_2 - \hat{z}_2) + \beta_{34} (s_2 - \hat{s}_2),
\]
\[
\psi_4 = \beta_{43} (z_2 - \hat{z}_2) + \beta_{44} (s_2 - \hat{s}_2),
\]
\[
\psi_5 = \beta_{55} (z_3 - \hat{z}_3) + \beta_{56} (s_3 - \hat{s}_3),
\]
\[
\psi_6 = \beta_{65} (z_3 - \hat{z}_3) + \beta_{66} (s_3 - \hat{s}_3),
\]
(22)
\[
\beta_{33} \beta_{44} - \beta_{34} \beta_{43} \neq 0;
\]
\[
\beta_{55} \beta_{66} - \beta_{56} \beta_{65} \neq 0;
\]
\[
\beta_{ij} \neq 0.
\]
Assessments of state variables \( z_2, s_2 \), \( z_2, s_2 \), could be defined:
\[
\hat{z}_2 = f_3(x_2) + w_3, \quad \hat{s}_2 = f_4(x_2) + w_4,
\]
\[
\hat{z}_3 = f_5(x_3) + w_5, \quad \hat{s}_3 = f_6(x_3) + w_6.
\]
(23)
The macro variables (22) must be satisfy functional equations
\[
\psi_i(t) + L_i \psi_i = 0, \quad L_i > 0, \quad i = 3, ..., 6.
\]
(24)
With received equations, formed by putting (22) into (16), subjecting to model (11), we need to choose functions
\[
f_1(x_2), f_2(x_2), f_3(x_2), f_4(x_2), f_5(x_2), f_6(x_2), L_i, \quad i = 3, ..., 6
\]
so that expressions of observers will not consist in itself unobserved state variables. We choose:
f_3(x_3) = \frac{\beta_{34}^2 \beta_{41}^2 - \beta_{44}^2 \beta_{33}^2}{\beta_{34}} x_3 + \frac{\beta_{44}}{\beta_{44}}

f_4(x_4) = \left( \frac{\beta_{34}^2 \beta_{41}^2}{\beta_{34}} \beta_{44} \left( \beta_{53} \beta_{44} - \beta_{34} \beta_{43} \right) - \sigma_2^2 \right) x_4;

L_3 = -\frac{\beta_{53}}{\beta_{56}} > 0, L_4 = -\frac{\beta_{63}}{\beta_{66}} > 0

 Consequently observer equations are formed

\dot{z}_1(t) = \left( \frac{\beta_{53}}{\beta_{54}} \right)^2 + \sigma_3^2 + \frac{\beta_{63} \beta_{63}}{\beta_{64}} + \left( \frac{\beta_{63}}{\beta_{64}} \right)^2 x_5 +

+ \left( \frac{\beta_{53}}{\beta_{54}} \right)^2 \left( w_1 + a_1 u_1 - g \cos x_5 \right) + w_5;

\dot{z}_2(t) = \left( \frac{\beta_{55}}{\beta_{56}} \right)^2 + \sigma_3^2 + \frac{\beta_{55} \beta_{55}}{\beta_{56}} + \left( \frac{\beta_{55}}{\beta_{56}} \right)^2 x_5 +

+ \left( \frac{\beta_{55}}{\beta_{56}} \right)^2 \left( w_5 + a_5 u_5 \right) + w_5;

\dot{z}_3(t) = \left( \frac{\beta_{65}}{\beta_{66}} \right)^2 + \sigma_3^2 + \frac{\beta_{65} \beta_{65}}{\beta_{66}} + \left( \frac{\beta_{65}}{\beta_{66}} \right)^2 x_5 +

+ \left( \frac{\beta_{65}}{\beta_{66}} \right)^2 \left( w_5 + a_5 u_5 \right) + w_5;

\dot{z}_4(t) = \left( \frac{\beta_{65}^2 \beta_{45}^2 - \beta_{66}^2 \beta_{35}^2}{\beta_{66}} \right) x_5 + w_5,

\dot{z}_5(t) = \left( \frac{\beta_{65}^2 \beta_{45}^2 - \beta_{66}^2 \beta_{35}^2}{\beta_{66}} \right) x_5 + w_5,

\dot{z}_6(t) = \left( \frac{\beta_{65}^2 \beta_{45}^2 - \beta_{66}^2 \beta_{35}^2}{\beta_{66}} \right) x_5 + w_5;

\dot{z}_7(t) = \left( \frac{\beta_{65}^2 \beta_{45}^2 - \beta_{66}^2 \beta_{35}^2}{\beta_{66}} \right) x_5 + w_5;

Thus, combining equations (20) and (27), we obtain a nonlinear state observer for external disturbances in the form of harmonic wave. Note that unobserved variables z_1, z_2, z_3 in the synthesized controls (10) should be replaced by its estimates \dot{z}_1, \dot{z}_2, \dot{z}_3 (15) and (28).

4. SIMULATION

The simulation results of the closed-loop system (11) with the synthesized NEPO are shown in nine figures below. A computational simulation of the given closed system was conducted with following parameters of CO model:

The mass of CO m = 25000 kg; the inertia moments concerning mutually perpendicular CO axes I_x = 48000 kg.m^2; I_y = 150000 kg.m^2; I_z = 116000 kg.m^2;

In figures from 1 to figure 9, the simulation results of closed system (3), (10) with periodical disturbances are showed. Parameters of environment are: wave attitude \theta = 2\, m \,; a wave angle frequency \sigma = 1.39\, s^{-1} \,; wave length \lambda = 26\, m \,; a angle that CO meets a sea wave \xi = 30^0 \,; coefficients \mu_i = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix}^T.
In conducting simulation results showed that CO’s longitudinal motion control objectives are achieved. Using synthesized control laws could significantly improve a motion performance: decreasing pitch angle oscillation, angular rate fluctuations and CO’s gravity center oscillation. The observers estimate unobserved disturbances with high measurement accuracy (fig.7-fig.9).

Thus, using synergetic control theory enable to create new classes of aircraft amphibian motion control systems.

6. REFERENCES


Contact information
Phuong Nguyen (Mr.)– (+84) 906 685 961
Faculty of Information Technology
University of Technical Education HCMC
Email: phuongn@fit.hcmute.edu.vn