Optimization of Process Parameters and Fatigue Prediction for Flexure-Based Compliant Mechanism

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Abstract

This study focuses on analysis and optimization for a rectangular leaf flexure hinge. The analysis was performed by employing pseudo-rigid-body model (PRBM) and the principle of virtual work. The PRBM was validated by using the finite element analysis (FEA). The simultaneously multi-objective structural optimization, such as the maximum torque of torsional spring and the minimum stress at flexible pivot, carried out using the fuzzy logic based on Taguchi method. ANOVA was used to find the most significant parameter. This investigation also focuses on predicting the fatigue strength of flexure hinge. Kinematic and static equations were formulated. The FEA results are a good agreement with the PRBM. The optimal results indicated that the input horizontal force of $0.8896 \text{ N}$ and rotary angular of $0.4363 \text{ radian}$. An input rotational angle is the most significant parameter with contribution of $44.2544 \%$. The fatigue analysis determined that the mechanism could reach approximately one million life cycles before failure.

Keywords: Flexure Hinge, Pseudo-Rigid-Body Model, Optimization, Fuzzy Logic Based on Taguchi Method, Fatigue Strength.

1. Introduction

In recent years, the compliant mechanisms (CMs) are well-known that are the elastic mechanisms based on the elasticity of flexible beams or flexure hinges so that transfer translational and/or rotational motions into the translational and/or rotational motions. CMs also called as flexible, monolithic structures. So far, CMs are that there has much increasing attention in automotive, space and appliance due to more accurate, cleaner, less noisy, and most importantly, less expensive to manufacture and maintain than conventional devices. There have numerous applications of flexure hinges in both macro- and microscale as: MEMS, automotive industry, computers and fiber optics applications. For examples, including optical switches, miniature load cells, flexible mounts for imaging masks, load-sensitive resonators, gyroscopes, disc memory head positioners, wire bonding heads, microaccelerometers, and cantilevers for microscopy.

Traditional bearing is commonly utilized in machine design based on rolling elements where allow rotation between two rigid parts. They are easy to manufacture and assemble. However, because of friction between the contact surfaces, rolling-element bearing is much energy loss. Therefore, bearing usually need lubrication to reduce friction and remove heat. To overcome the disadvantages of conventional bearings, a flexure hinge with rectangular cross-section was used in this study. In present study, a flexure hinge is innovative design which has
the same functions of traditional bearing. Overcome limitations of traditional kinematic joints, flexure hinges take some advantages as: non-friction, no need lubrication, ease of fabrication, and no maintenance. There are however certain drawbacks in flexure-based compliant mechanisms: (1) there is no pure rotation due to flexure deformation is complicate and (2) relative rotation with rotary center is not fixed during motion. Another disadvantage is that flexure hinges are temperature-sensitive and therefore thermal changes will modify their physical dimensions. Consequently, the compliance/stiffness properties will affect the motion precision and repeatability. Under dynamic loading conditions CMs will introduce failures like the fatigue that approaches to fracture overall structure. As a result, the desired structure of flexure hinge will be optimized in this study to meet the user-specified motion requirements such as the large enough deflection/displacement required while the stresses below an allowable maximum stress.

Several approaches have increased to address CMs by applying optimal design and synthesis procedures. Frecker et al. [1] presented topology of CMs with multiple outputs. A pseudo-rigid-body (PRBM) method used to analyze nonlinear large displacement performed by Pei et al. [2] in 2010. According to Tanki and Söylemez [3] analyzed and designed of an underactuated compliant variable stroke mechanism by employing PRBM. In 2003, Lobontiu and Garcia [4] formulated an analytical method for displacement and stiffness calculations of planar compliant mechanisms with single-axis flexure hinges. The procedure is based on the strain energy and Castigliano’s displacement theorem and produces closed-form equations.

In addition to the optimal result of the separately individual objectives are usually conflicts each other in the optimization process. In past decades, there have increasing interested optimal procedures such as Taguchi method, genetic algorithm (Gas), fuzzy logic controller (FLC) that used in almost industrial fields. Unfortunately, Taguchi method is only capable to optimize individual objectives separately; in contrast to single objective optimization, the suggested approaches like GAs, FLC, FLC combined GAs and/or Taguchi method are likely to optimize multi-objectives simultaneously. In such cases, in order to solve the conflict optimal issues between multi-objectives, the fuzzy logic based on Taguchi method (FLTM) has been utilized like the most effective tool recently. Eşme [5] optimized the welding parameters using Taguchi method. Lin and Kuo [6] optimized structural design of drawing dies using FLTM in 2010. Another study by Hsiang et al. [7] found optimal process parameters that maximize MRPI based on FLTM for hot extrusion of AZ31 and AZ61 magnesium alloy bicycle carriers. Sze’kely and Szalay [8] suggested a systematic design procedure for two-fingered microgripper with flexure hinges. According to Venanzi et al. [9] presented an iterative technique to perform the non-linear position analysis of planar CMs. In 2008, N.T. Pavlovic’ and N.D. Pavlovic [10] performed a design of the compliant mechanism being capable to realize axial link translation. In 2005, the compliant contact-aided revolute joint, a planar mechanism capable of performing the functions of a bearing and a spring designed using PRBM by Cannon and Howell [11]. Khatait et al. [12] analyzed the dynamics of a flapping compliant mechanism consisting of two flexure hinges using minimum torque approach. In 2005 as well, optimization of multi-objective topology of CMs-continuum structures conducted by Luo et al. [13] using a density interpolation scheme, the rational approximation of material properties method, and a globally convergent version of the method of moving asymptotes. Huang et al. [14] designed and fabricated a microgripper with a topology optimal compliant mechanism in 2009. Ugwuoke et al. [15] investigated dynamic behavior of compliant slider mechanism using the PRBM in 2009 as well. Another study also in 2009 by Boyle et al. [16] gave a mathematical dynamic model for compliant, constant-force compression mechanisms using PRBM. Dado [17] presented a variable parametric
PRBM for large-deflection beams with end loads. More recent work in 2013, Osakue et al. [18] reported a probabilistic design approach for the Gerber bending fatigue failure rule using sensitivity-based analysis. Also in 2013, Dirksen et al. [19] studied incorporation of flexural hinge fatigue-life cycle criteria into the topological design of compliant small-scale devices.

This investigation first concentrates on design, analysis of the fatigue strength of flexure hinge. Next, finite element analysis (FEA) was used to validate for PRBM theory. After that, the study presents the application of the FLTM to multi-objective structural optimization of flexure hinge in CMs. The maximum torque of torsional spring and the minimum stress are two objective functions were formulated by PRBM that required to be optimized simultaneously to meet user-desired requirements.

2. Modeling of Flexure Based Compliant Mechanism

Based on the results of previous study by Dirksen et al. [20], a rectangular cross-section flexure hinges have low bending stiffness with very high rotational deflection while circular cross-section flexure hinges obtain medium bending stiffness with high rotational deflection and parabolic cross-section flexure hinges achieve high bending stiffness with low rotational deflection. A rectangular leaf flexure hinge is selected to achieve single-axis flexure hinge and large deflection with low bending stiffness. Figure 1 shows a model of flexure based CMs, including a rigid link and a rectangular cross-sectional flexure hinge.

To analyze nonlinear-large deflection, the use of the Bernoulli-Euler theory is very difficult; as a recent result, the theory PRBM [21] as an effectively alternative approach that has utilized to design and analyze nonlinear-large deflection by modeling a flexure hinge into a rigid link with a torsional spring located rotational center as in Fig.2a. A deflected model after a mobile link rotates a specific angular, Θ, is presented in Fig.2b.

![Fig. 1. Model of flexure based CMs](image-url)
3. Kinematic and Static Analysis of Flexure Based Compliant Mechanism

A total length of rigid body which composes of mobile link connected with link AB is equal to $L = 127\, \text{mm}$, a length of flexure hinge is $l = 12.7\, \text{mm}$. A compliant rotary joint is rectangular cross-sectional flexure hinge with the out of plane thickness, $w = 22.86\, \text{mm}$ the in of plane thickness, $h = 0.762\, \text{mm}$. Load conditions include a horizontal force acts on a mobile body, has the range from $0.8896\, \text{N}$ to $88.9644\, \text{N}$. This flexure hinge made of polycarbonate with a Young’s modulus $E = 2000\, \text{MPa}$ with ultimate tensile strength equals to 70 MPa, yield strength equals 65 MPa. The chosen angular, $\Theta$, is generalized coordinate for this issue. In case study, this paper will analyze the relationship between non-linear displacement and applied forces; the paper concentrates on the maximum stress occurs at fixed end of flexible pivot as well. The virtual work [21] used for analyzing kinematics and statics as follows:

A horizontal coordinate of a horizontal deflection, $b$, is

$$ b = \left( L + \frac{l}{2} \right) \sin \Theta $$

(1)

The virtual horizontal translational displacement, $\delta b$, is differential of position vector is

$$ \delta b = \left( L + \frac{l}{2} \right) \cos \Theta \delta \Theta $$

(2)

Torque of flexure hinge required to deflect a torsional spring through an angular of $\Theta$ is

$$ T = -K \Theta $$

where $K$ is the spring constant

(3)
Inertia torque of flexure hinge cross section is

\[ I = \frac{wh^3}{12} \]  

The total virtual work for the pseudo-rigid-body model is

\[ \delta W = -F \left( L + \frac{l}{2} \right) \partial \Theta \cos \Theta + T \partial \Theta \]  

Application of total virtual work is equal to zero, the torque of spring at pin joint results in

\[ T = F \left( L + \frac{l}{2} \right) \cos \Theta \]  

The horizontal deflection calculated in terms of \( L, l, \) and vertical coordinate of end mobile, \( a, \) is

\[ \delta_x = L + l - a \]  

The maximum stress, \( \sigma_{\text{max}} \), occurs at the fixed end and calculated as follows:

\[ \sigma_{\text{max}} = \frac{6Fa}{wh^2} \]  

where \( a \) is the vertical direction coordinate of the beam’s end as

\[ a = \frac{l}{2} + \left( L + \frac{l}{2} \right) \cos \Theta \]  

As a result

\[ \sigma_{\text{max}} = \frac{6F \left[ \frac{l}{2} + \left( L + \frac{l}{2} \right) \cos \Theta \right]}{wh^2} \]  

The force \( F \) and angular deflection has the following relationship
\[ F = \frac{K\Theta}{(L + l/2)\sin(\pi/2 - \Theta)} \]  \hspace{1cm} (12)

The torque, \( T \), is proportional with the angular displacement, \( \delta\Theta \), \( T = K\Theta \) in magnitude with \( K \) is the stiffness constant of spring, i.e., the larger the value of torque is, larger the value of angular displacement is. From Eq.11 the maximum stress, \( \sigma_{\text{max}} \), occurs at the fixed end and was calculated terms of two variables composed of \( F \) and \( \Theta \). Fig. 3 derives the linear relationship between the max stress and the horizontal displacement that considers an approximately linear equation as follows:

\[ y = 2020x + 1 \]  \hspace{1cm} (13)

![Graph](image)

Fig. 3. A relationship between the maximum stress versus the horizontal displacement

### 4. Optimization of Flexure Based Compliant Mechanism

#### 4.1 Formulation of the optimization problem

In this section, this study determines the optimal process parameters that influence the structure of a flexure hinge by simultaneous optimizing the torque of torsional spring and stress at fixed end of flexure hinge. The optimization problem for proposed CMs is formulated as follows:

Maximize the torque of torsional spring:

\[ f_1 (F, \Theta) = F \left( L + \frac{l}{2} \right) \cos \Theta \]  \hspace{1cm} (14)
Minimize the stress at fixed end of flexure hinge:

\[
f_2(F, \Theta) = \frac{6F\left[\frac{l}{2} + \left(\frac{L + l}{2}\right) \cos \Theta\right]}{wh^2}
\]  

subject to the constraints:

\[
\begin{cases}
48.9304N \leq F \leq 88.9644N \\
0.0872\text{radian} \leq \Theta \leq 0.7853\text{radian}
\end{cases}
\]  

Where \(F\) is an external force and \(\Theta\) is the rotational angle that must determine to satisfy two objective functions.

### 4.2 Optimal Procedure

In order to find optimal process parameter values based on a single quality characteristic, the Taguchi method is one of the most significant tools because it is an efficient experimental method, and only requires a small number of experiments to measure the quality and analysis of the optimal process. However, optimal results obtained using different quality characteristics always contradict each other. As a result, in an attempt to improve this contradictory problem, fuzzy logic combined with the Taguchi method (FLTM) was utilized in this paper to find the combination of process parameters that optimize the multi-response performance index (MRPI). The flow chart structure of the fuzzy logic controller coupled with the Taguchi method used in the study is shown in Fig. 4.
4.2.1. Taguchi method

Taguchi method applications are concerned with the optimization of a single performance characteristic. The Taguchi method uses a special design of orthogonal arrays to study an entire parameter space with only a small number of experiments. The experimental results are then transformed into a signal-to-noise (S/N) ratio. The S/N ratio can be used to measure performance characteristics deviating from the desired values. Usually, there are three categories of the performance characteristics in the analysis of the S/N ratio: the lower-the-better, the higher-the-better and the nominal-the-better. In this study, L9 orthogonal array experiment is used with the two right columns are ignored because there are two parameters and their three levels. To obtain optimal motion performance, maximum torque of torsional spring and the minimum stress at flexible pivot are desired. Therefore, the higher-the-better the torque of torsional spring and the lower-the-better stress should be selected. After determining the orthogonal array experiment and the number of parameters levels, this research performs calculation for S/N of the torque of torsional spring and the stress of flexible hinge as the following equations briefly describe:

The higher-the-better torque of torsional spring is:

$$ S / N_L = -10 \log \left( \frac{1}{n} \sum_{i=1}^{n} \frac{1}{y_i^2} \right) $$

(17)
The lower-the-better maximum stress is:

\[
S / N_\delta = -10 \log \left( \frac{1}{n} \sum_{i=1}^{n} y_i^2 \right)
\]  

where \( y \) is the observed data.

To consider the two different performance characteristics in the Taguchi method, the S/N ratios corresponding to the torque of torsional spring and stress at fixed end of flexure hinge are two inputs processed by the fuzzy logic control in order to find out optimal parameter values.

### 4.2.2 Fuzzy Logic Based on Taguchi Method

Using fuzzy logic control, the optimization of multiple performance characteristics can be transformed into the optimization of a single performance index. Thus, the proposed method is the integration of fuzzy logic control with the Taguchi method; they are used to simultaneously achieve the optimization of multiple performance characteristics.

A fuzzy logic unit comprises a fuzzifier, membership functions, a fuzzy rule base, an inference engine and a defuzzifier. First, the fuzzifier uses member functions to fuzzify the signal-to-noise ratios. Next, the inference engine performs fuzzy reasoning on fuzzy rules to generate a fuzzy value. Finally, the defuzzifier converts the fuzzy value into a multi-response performance index. The structure of the two-input-one-output fuzzy logic is shown in Figure 5. In the following, the concept of fuzzy reasoning is briefly described, based on the two-input-one-output fuzzy logic unit. The fuzzy rule base consists of a group of if-then control rules, with two inputs, \( x_1 \) and \( x_2 \), and one output, \( y \).

![Fig. 5. Structure of the two-input-one-output fuzzy logic control](image)

An orthogonal array, the S/N ratio and MRPI are used to study the performance characteristics of this mechanism. Matlab 7.1 software is used to solve the roots of nonlinear equations and support for fuzzy logic. Regardless of the category of the performance characteristic, a larger S/N ratio corresponds to a better performance characteristic. As a result, the optimal level of the process parameters is the level with the highest
S/N ratio. The loss function corresponding to each performance characteristic is fuzzified; then an MRPI is achieved through fuzzy reasoning using fuzzy rules.

4.3. Results and Discussions

According to the flow chat structure in Fig. 4 an optimal process for flexure hinge mechanism is performed using FLTM. This research formulated nine fuzzy rules in Table 1 that are directly based on the fact that the larger the S/N ratio is, the better the performance characteristic in fuzzy logic controller as presented.

The value of input horizontal force named A has the range from \(48.9304 \text{N} \leq F \leq 88.9644 \text{N}\), while the value of rotary angle of this mechanism has the range from \(0.0872 \text{rad} \leq \Theta \leq 0.7853 \text{rad}\); and the value of each of these process parameters was divided into three levels as shown in Table 2. Based on Taguchi method, the L9 orthogonal array with nine experiments and detailed values of each level was given in Table 3. The higher-the-better the torque of torsional spring and the lower-the-better stress should be selected in this study. After that, the membership functions were created by using Matlab software as presented in Fig. 6. The membership function of the S/N ratio of the torque of torsional spring has the range from 0 to 1. The membership function of the S/N ratio for the stress has the range from 0 to 1 as well. The membership functions can be adjusted to find real optimal parameters value. Both membership functions are trapezoidal shapes. Meanwhile, the membership functions of MRPI have triangle shapes and the range from 0 to 1. Figure 6 also presents the range of S/N ratio of the torque of torsional spring from -60 to -40, the range of the S/N ratio for the stress from -160 to -60, and the range of MRPI from 0 to 1. The results of S/N ratio of torque of torsional spring, the S/N ratio for the stress, and the MRPI were calculated in Table 4. Next, the mean of MRPI of input horizontal force A and rotary angular B at each of their levels was calculated in Table 5. The larger the MRPI is, the smaller the variance of performance characteristics around the desired value. Based on Fig. 7 the predicted optimal parameter levels are input horizontal force at level 1 of 0.8896 \(\text{N}\) and rotary angle at level 2 of 0.4363 radian.

### Table 1. Fuzzy rules

<table>
<thead>
<tr>
<th>MRPI</th>
<th>S/N ratio of stress (\sigma)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Small</td>
</tr>
<tr>
<td>Small</td>
<td>Very small</td>
</tr>
<tr>
<td>Medium</td>
<td>Small</td>
</tr>
<tr>
<td>Large</td>
<td>Medium</td>
</tr>
</tbody>
</table>

### Table 2. The values of process parameters and their levels

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Range</th>
<th>Unit</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Horizontal force F</td>
<td>0.8896-88.9644</td>
<td>N</td>
<td>0.8896</td>
<td>48.9304</td>
<td>88.9644</td>
</tr>
<tr>
<td>B</td>
<td>Rotational angular (\Theta)</td>
<td>0.0872-0.7853</td>
<td>Radian</td>
<td>0.0872</td>
<td>0.4363</td>
<td>0.7853</td>
</tr>
</tbody>
</table>
Table 3. Nine trials with detailed values

<table>
<thead>
<tr>
<th>Experiment No.</th>
<th>A, Horizontal force (N)</th>
<th>B, Rotational angular (radian)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8896</td>
<td>0.0872</td>
</tr>
<tr>
<td>2</td>
<td>0.8896</td>
<td>0.4363</td>
</tr>
<tr>
<td>3</td>
<td>0.8896</td>
<td>0.7853</td>
</tr>
<tr>
<td>4</td>
<td>48.9304</td>
<td>0.0872</td>
</tr>
<tr>
<td>5</td>
<td>48.9304</td>
<td>0.4363</td>
</tr>
<tr>
<td>6</td>
<td>48.9304</td>
<td>0.7853</td>
</tr>
<tr>
<td>7</td>
<td>88.9644</td>
<td>0.0872</td>
</tr>
<tr>
<td>8</td>
<td>88.9644</td>
<td>0.4363</td>
</tr>
<tr>
<td>9</td>
<td>88.9644</td>
<td>0.7853</td>
</tr>
</tbody>
</table>

Table 4. Results for S/N ratio and the MRPI

<table>
<thead>
<tr>
<th>Experiment No.</th>
<th>Torque of torsional spring T (Nm)</th>
<th>S/N ratio (dB) of Torque T</th>
<th>Stress σ (MPa)</th>
<th>S/N ratio (dB) of Stress</th>
<th>MRPI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0336</td>
<td>-67.8646</td>
<td>17.765031</td>
<td>-57.5446</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0.1175</td>
<td>-42.8263</td>
<td>55.707570</td>
<td>-80.4023</td>
<td>0.514</td>
</tr>
<tr>
<td>3</td>
<td>0.0623</td>
<td>-55.5159</td>
<td>30.724417</td>
<td>-68.5012</td>
<td>0.5</td>
</tr>
<tr>
<td>4</td>
<td>0.8508</td>
<td>-3.2316</td>
<td>977.056054</td>
<td>-137.6909</td>
<td>0.433</td>
</tr>
<tr>
<td>5</td>
<td>6.4674</td>
<td>37.3355</td>
<td>3063.892240</td>
<td>-160.5488</td>
<td>0.5</td>
</tr>
<tr>
<td>6</td>
<td>3.4276</td>
<td>24.6372</td>
<td>1689.836061</td>
<td>-148.6477</td>
<td>0.499</td>
</tr>
<tr>
<td>7</td>
<td>3.3651</td>
<td>24.2692</td>
<td>1776.503160</td>
<td>-149.6480</td>
<td>0.487</td>
</tr>
<tr>
<td>8</td>
<td>11.7590</td>
<td>49.2924</td>
<td>5570.757039</td>
<td>-172.5057</td>
<td>0.5</td>
</tr>
<tr>
<td>9</td>
<td>6.2321</td>
<td>36.5943</td>
<td>3072.441739</td>
<td>-160.6046</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 5. MRPI for input parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Input parameter</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Max-Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Horizontal force</td>
<td>0.505</td>
<td>0.47</td>
<td>0.496</td>
<td>0.035</td>
</tr>
<tr>
<td>B</td>
<td>Rotational angular</td>
<td>0.47</td>
<td>0.505</td>
<td>0.499</td>
<td>0.035</td>
</tr>
</tbody>
</table>

Total of MRPI = 0.498
Fig. 6. Membership functions of torque and stress and MRPI
The optimal results were then compared with yield strength of used material (polycarbonate), it showed that optimized maximum stress was 55.7 MPa that is much less than value of yield strength 65 MPa in second experiment with force of 0.8896 N and rotary angular of 0.4363 radian. The utilized mechanism in this study is an elastic mechanism, thus the comparison meets user requirements. An optimal process ends.

An analysis of variance (ANOVA) is performed to identify the process parameter that is statistically significant affecting performance characteristics. Table 6 indicates that input rotational angular is the most significant parameter affecting structure of flexure hinge with contribution of 44.2544%.

Table 6. Results of the analysis of variance

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameters</th>
<th>Degree of freedom</th>
<th>Sum of squares</th>
<th>Mean square</th>
<th>F</th>
<th>Contribution(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Horizontal force</td>
<td>2</td>
<td>0.00016822</td>
<td>8.4111e-005</td>
<td>4.6371</td>
<td>26.5482</td>
</tr>
<tr>
<td>B</td>
<td>Horizontal force</td>
<td>2</td>
<td>0.00025622</td>
<td>1.2811e-004</td>
<td>7.0628</td>
<td>44.2544</td>
</tr>
<tr>
<td></td>
<td>rotational angular</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td></td>
<td>4</td>
<td>0.000072556</td>
<td>1.8139e-005</td>
<td>29.1974</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>8</td>
<td>0.00049700</td>
<td>2.3036e-004</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

5. A Validation of PRBM Using FEA

A non-linear finite element analysis (FEA) in ANSYS is used to evaluate the theory of PRBM of the mechanism. This study utilized hex dominant mesh method with quadrilateral-geometric meshed elements. Assuming that a horizontal acted force value is 0.8896 N. An input rotational angular has a range from 0.0872 radian to 0.7853 radian. After that running on Matlab 7.1, a solution for finding out the maximum stress at flexible pivot with PRBM from Eq.12 using non-linear programming with the subject to of force $F = 0.8896N$ and rotational angular value $0.0872 radians \leq \Theta \leq 0.7853 radians$. The results of PRBM found that a maximum stress is 55.7 MPa; while using FEA, the maximum stress is 54.4 N (Fig. 8). As a result, the PRBM and FEA prediction are very close, and have a maximum difference of only 0.01202%. A comparison between PRBM and FEA is illustrated in more detail in Fig. 8 at three points on these figures are that approximately values of the maximum stresses. This variation is most likely due to the variation the correction of mesh density of elements and in ANSYS a flexure hinge mechanism is taken into account like volume bodies, in contrast to a solution uses PRBM that based on line geometry of bodies. The results of FEA are a good agreement theory PRBM with FEA.
6. Fatigue Analysis

The fatigue failure is always the most important one of issues in design phase that needs to consider in machine design. In this study, the stress is fluctuating stress with nonzero mean stress as shown in Fig. 9. Thus, a rectangular leaf flexure hinge mechanism will introduce to a fatigue failure during cycle-repeated motion. This paper used the modified Goodman criterion [21] to predicting fatigue strength of a flexure hinge. The modified Goodman diagram is one of the common procedures for calculating and predicting fatigue strength for polymer materials.

![Fatigue Analysis Diagram](image)

The fluctuating stresses are more general loading condition in compliant mechanisms. Therefore, the theory of the modified Goodman is used commonly in this case for polymer materials. The $\sigma_m$ and $\sigma_a$ are the mean stress and the amplitude (alternative) stress, respectively. These stresses are formulated in terms of the maximum stress.
as the following.

\[
\sigma_m = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2}
\]

(19)

\[
\sigma_a = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2}
\]

(20)

The safety factor, SF, for the modified Goodman is expressed in terms of the endurance limit, Se, and the fatigue strength, Sf as follows:

In terms of the endurance limit, modified Goodman equation is

\[
\frac{1}{SF} = \frac{\sigma_a + \sigma_m}{S_e} + \frac{\sigma_m}{S_{ul}}
\]

(21)

In terms of the fatigue strength, modified Goodman equation is

\[
\frac{1}{SF} = \frac{\sigma_a + \sigma_m}{S_f} + \frac{\sigma_m}{S_{ul}}
\]

(22)

where Sut, is the ultimate strength of a given material

At yielding, safety factor is also calculated as equation below:

\[
SF = \frac{S_y}{\sigma_m + \sigma_a}
\]

(23)

where Sy is the yield strength of a given material.

The stress fluctuates from zero to maximum value \(\sigma_{\text{max}} = 55.7 \text{ MPa}\), so the above modified Goodman equations are utilized to estimate the fatigue strength in this study.

With optimized maximum stress is equal to 55.7 MPa, the yield strength and the ultimate strength of polycarbonate are 65 MPa and 70 MPa, respectively. Form Eqs. 19 and 20, the mean stress and alternative stress results in: \(\sigma_m = 27.85 \text{ MPa}, \sigma_a = 27.85 \text{ MPa}, SF = 1, S_e = 59.05 \text{ MPa}, S_f = 59.05 \text{ MPa}\). The result shows that \(\sigma_{\text{max}}\) is much less than modified fatigue strength.

Next, the fatigue cycle life is calculated as follows:
\[
\log N = \frac{1}{b_f} \log \frac{S}{a_f}
\]  
(24)

Where \(a_f\) and \(b_f\) are cure-fit parameters. For bending load \(c_f = 0.9\)

In this work, as bending load results in

\[
a_f = \frac{(c_f S_{ut})^2}{S_e}
\]  
(25)

\[
a_f = 67.2
\]

\[
b_f = \frac{-1}{3} \log \frac{c_f S_{ut}}{S_e}
\]  
(26)

\[
b_f = \frac{-1}{3} \log \frac{0.9 * 10152}{9104} = -0.009
\]

The maximum number of life cycles before fatigue failure is \(N = 10^6\).

7. Conclusion

This paper presents a design and analysis of a rectangular cross-section flexure hinge that improved from a traditional bearing by applying PRBM and virtual work. A FEA based on ANSYS to evaluate PRBM. Next, this study focuses multi-objective structural optimization of a flexure hinge by employing the fuzzy logic based on Taguchi method. The maximum angular deflection/displacement (or maximum torque of torsional spring at flexible pivot) and the minimum stress at flexible pivot are the two objectives that need to optimize simultaneously. ANOVA was used to find the most significant parameter. This investigation also concentrates on predicting the fatigue strength of this hinge with modified Goodman criterion.

The results of FEA are a good agreement with the PRBM. The results found that the input horizontal force at level 1 of 0.8896 N and rotary angle at level 2 of 0.4363 radian are favorable parameters for a flexure hinge. The result also indicated that input rotational angular is the most significant parameter affecting structure of flexure hinge with contribution of 44.2544 %. The result of a fatigue analysis determined that the mechanism could reach approximately 1 million cycles before failure. The design, analysis, and optimization approaches introduced in this study are also applicable for other similar flexure hinges. Future work will conclude an investigation into the location of the center of rotation and motion of rectangular leaf flexure hinge and the confirmation analytical model by experiments.

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References


